Improving the Forecasting Properties of the Lee-Carter Model

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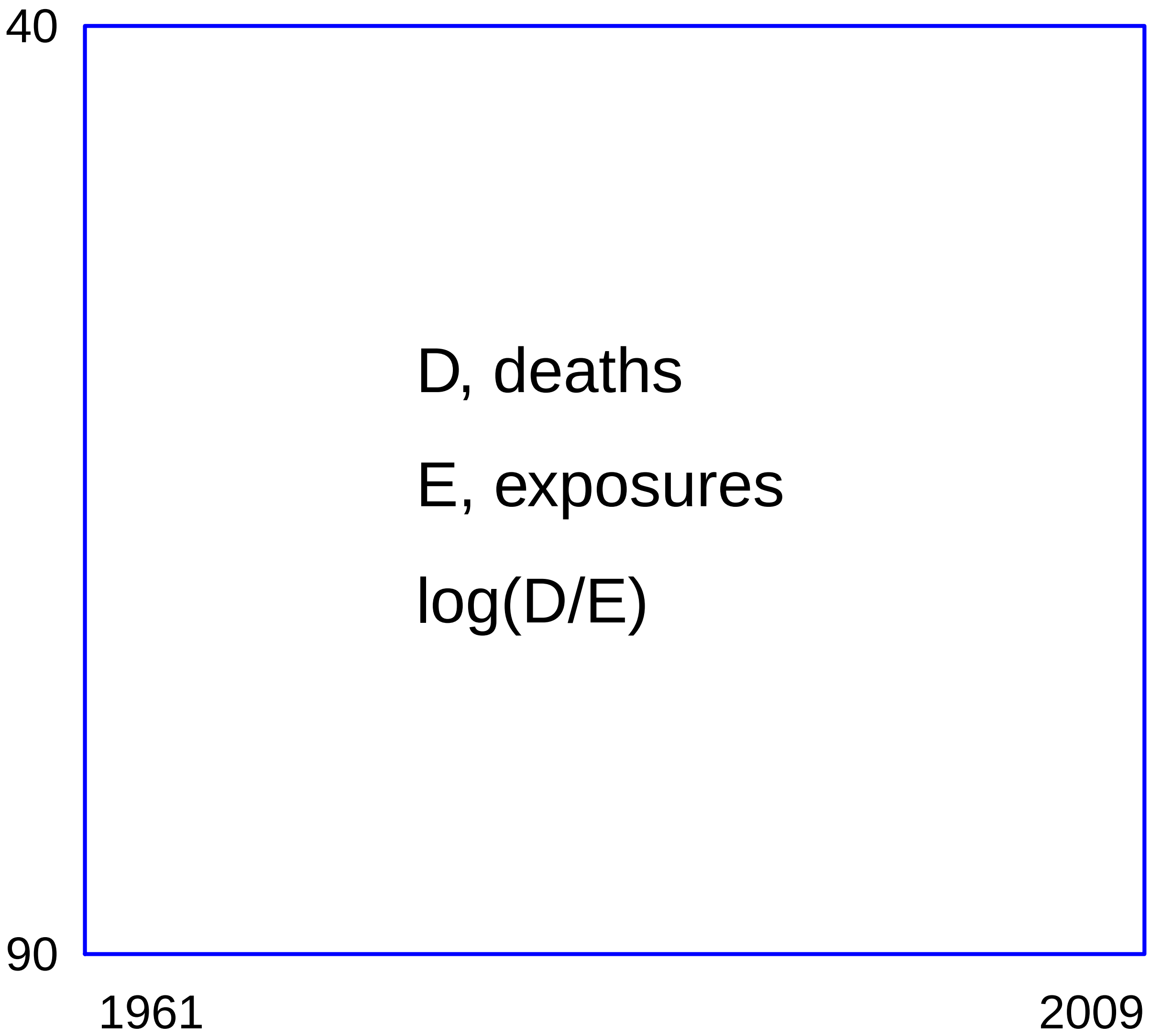
London
19th April 2012

General Themes

- Model fit
- Forecast properties
- Tension between model fit and forecast properties.
- More general problem in statistics: model choice. Must balance
  - Fit to data $\Rightarrow$ more complex model.
  - Clarity of signal $\Rightarrow$ less complex model.

UK male mortality data



Lee-Carter model

$$\log(\mu_{i,j}) = \alpha_i + \beta_i \kappa_j$$

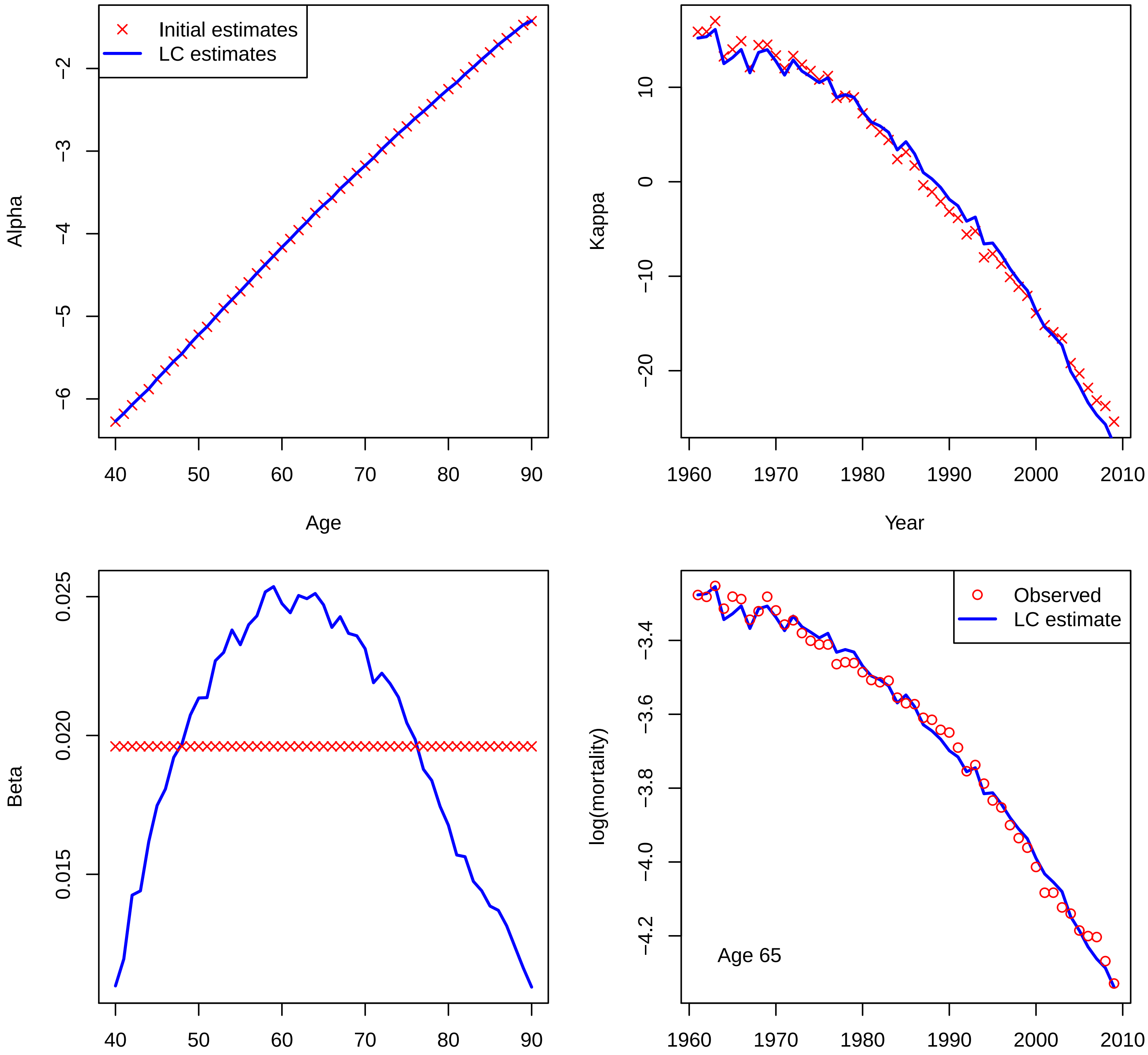
$$\sum \kappa_j = 0, \quad \sum \beta_i = 1$$

Note: The Projections Toolkit uses a different scale constraint, $\sum \kappa_j^2 = 1$.
We can move from one set of estimates to the other by a simple scale change.
The choice of scale constraint has no effect on the forecast.

Next slide displays estimates of $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and $\boldsymbol{\kappa}$. The $\times$ corresponds to initial estimates set as follows:

- $\boldsymbol{\alpha}$: row means of $\log\left(\boldsymbol{D}/\boldsymbol{E}\right)$.
- $\boldsymbol{\kappa}$: column means of $\log\left(\boldsymbol{D}/\boldsymbol{E}\right)$, centred and scaled.
- $\boldsymbol{\beta}$: values of β_i all set to $1/n_x$ where n_x is the number of ages.

Note: The Lee-Carter estimates exhibit stability: the introduction of $\boldsymbol{\beta}$ has little effect on the initial estimates of $\boldsymbol{\alpha}$ and $\boldsymbol{\kappa}$.



Initial estimates
LC estimates
Alpha
Age
Kappa
Year
Beta
Observed
LC estimate
log(mortality)
Age 65

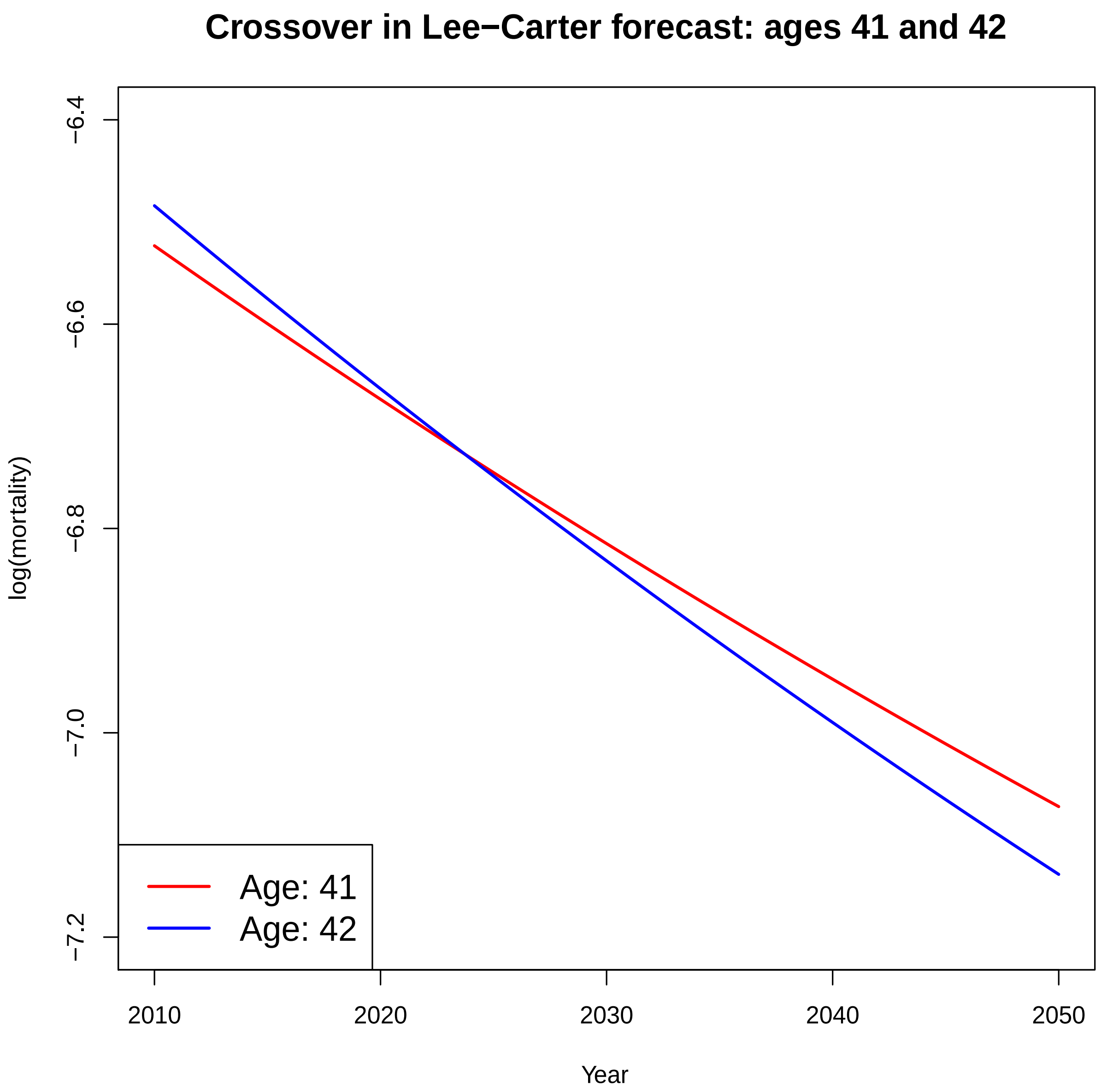
Crossovers

Forecasts with the Lee-Carter model can suffer from

- Crossovers: forecast rates which crossover at adjacent ages.

Small populations (eg, CMI) are particularly prone to these problems but large populations (eg, male UK) are not immune.

Example: ARIMA(1,1,1) forecast at ages 41 and 42 shown over.

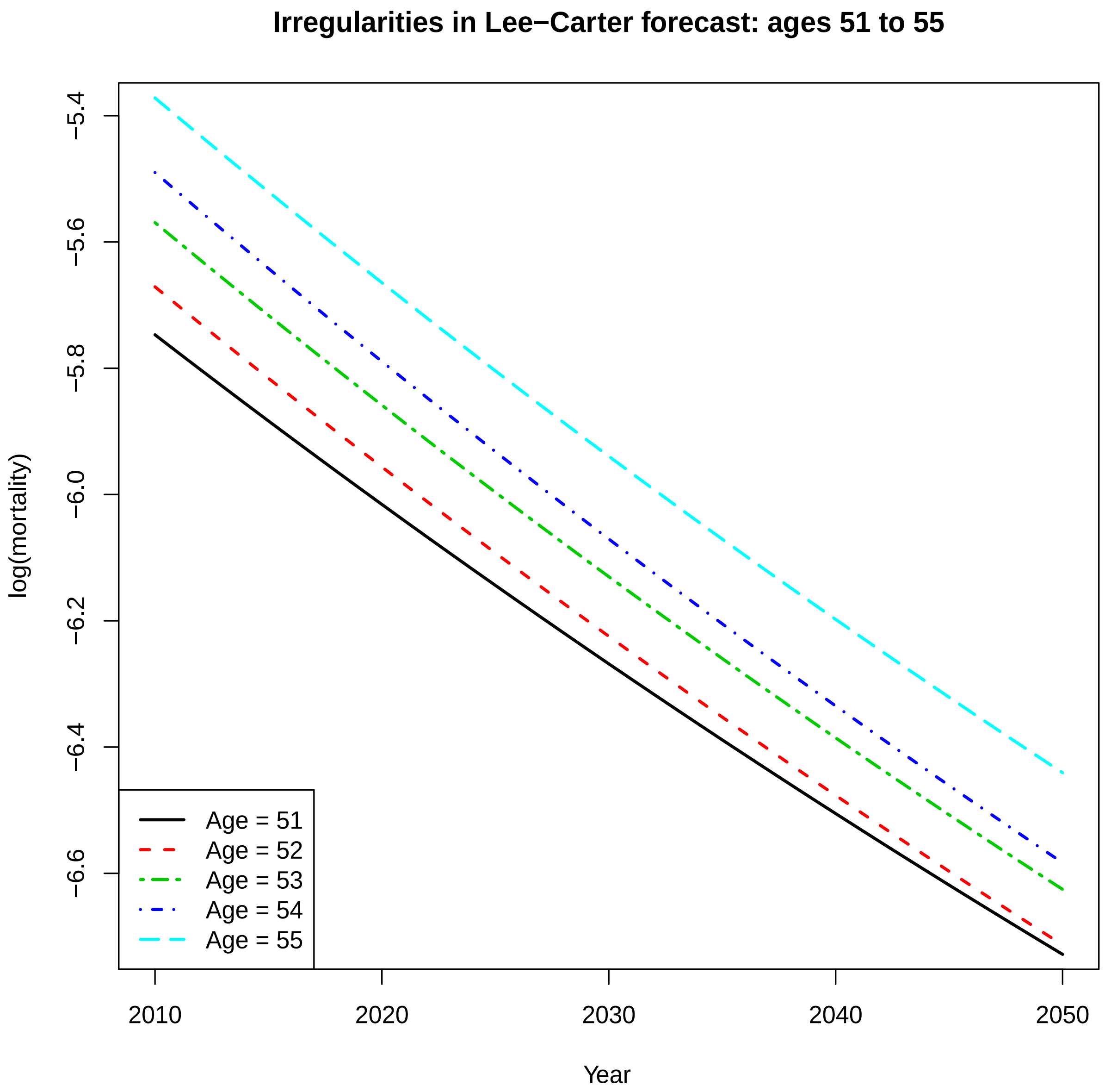

Crossover in Lee−Carter forecast: ages 41 and 42
log(mortality)
−6.4
−6.6
−6.8
−7.0
−7.2
2010
2020
2030
2040
2050
Year
Age: 41
Age: 42

Irregularities

Forecasts with the Lee-Carter model also suffer from

- **Irregularity:** forecast rates which have irregular gaps between adjacent ages.

Example: ARIMA(1,1,1) forecast with irregular gaps from ages 51 to 55.



Irregularities in Lee−Carter forecast: ages 51 to 55
log(mortality)
−5.4
−5.6
−5.8
−6.0
−6.2
−6.4
−6.6
Year
2010
2020
2030
2040
2050
Age = 51
Age = 52
Age = 53
Age = 54
Age = 55

Delwarde, Denuit & Eilers

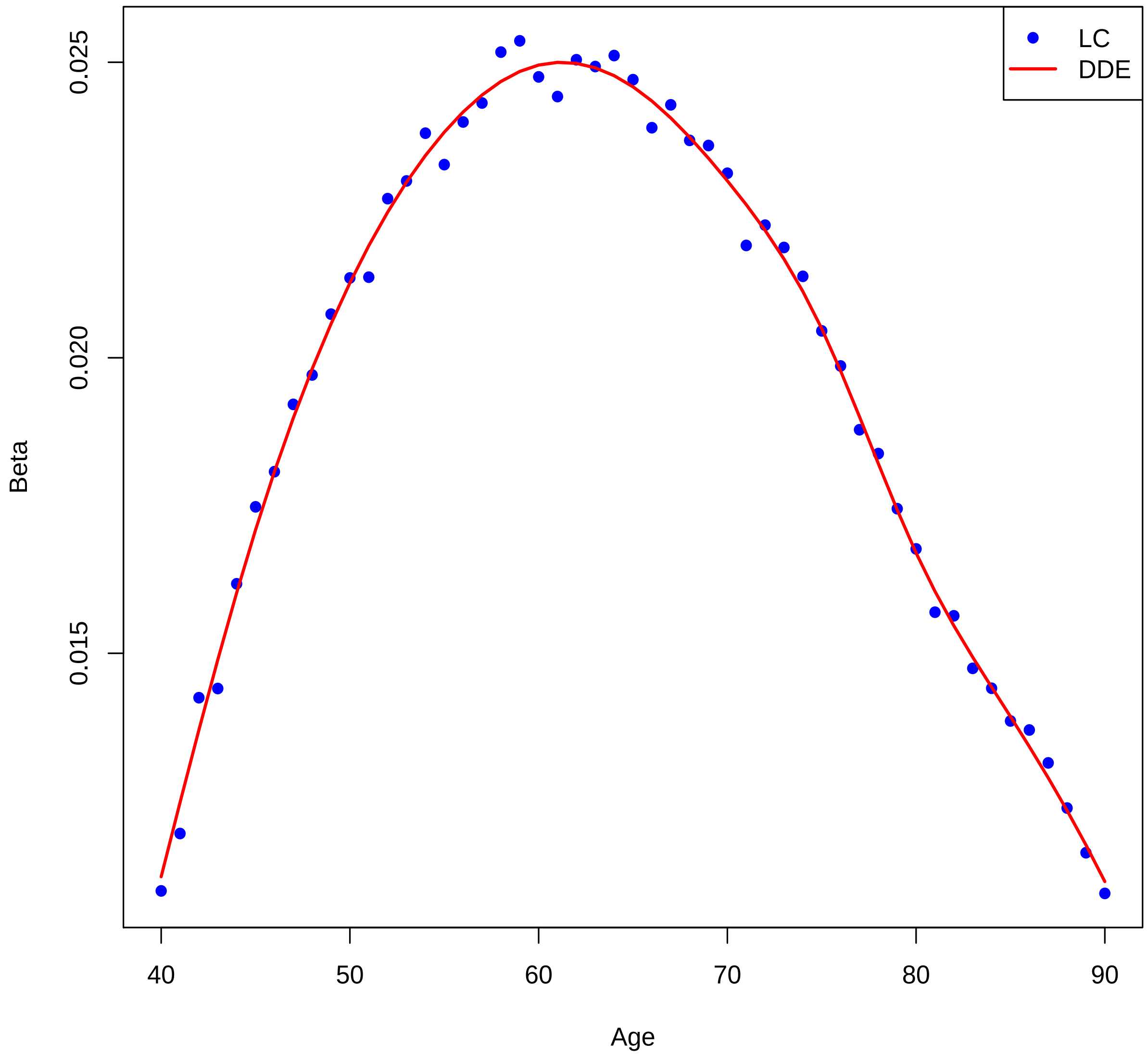
Delwarde, Denuit & Eilers (2007) identified

- Irregularities in the β_i as the reason for crossovers and irregularities in the forecast.

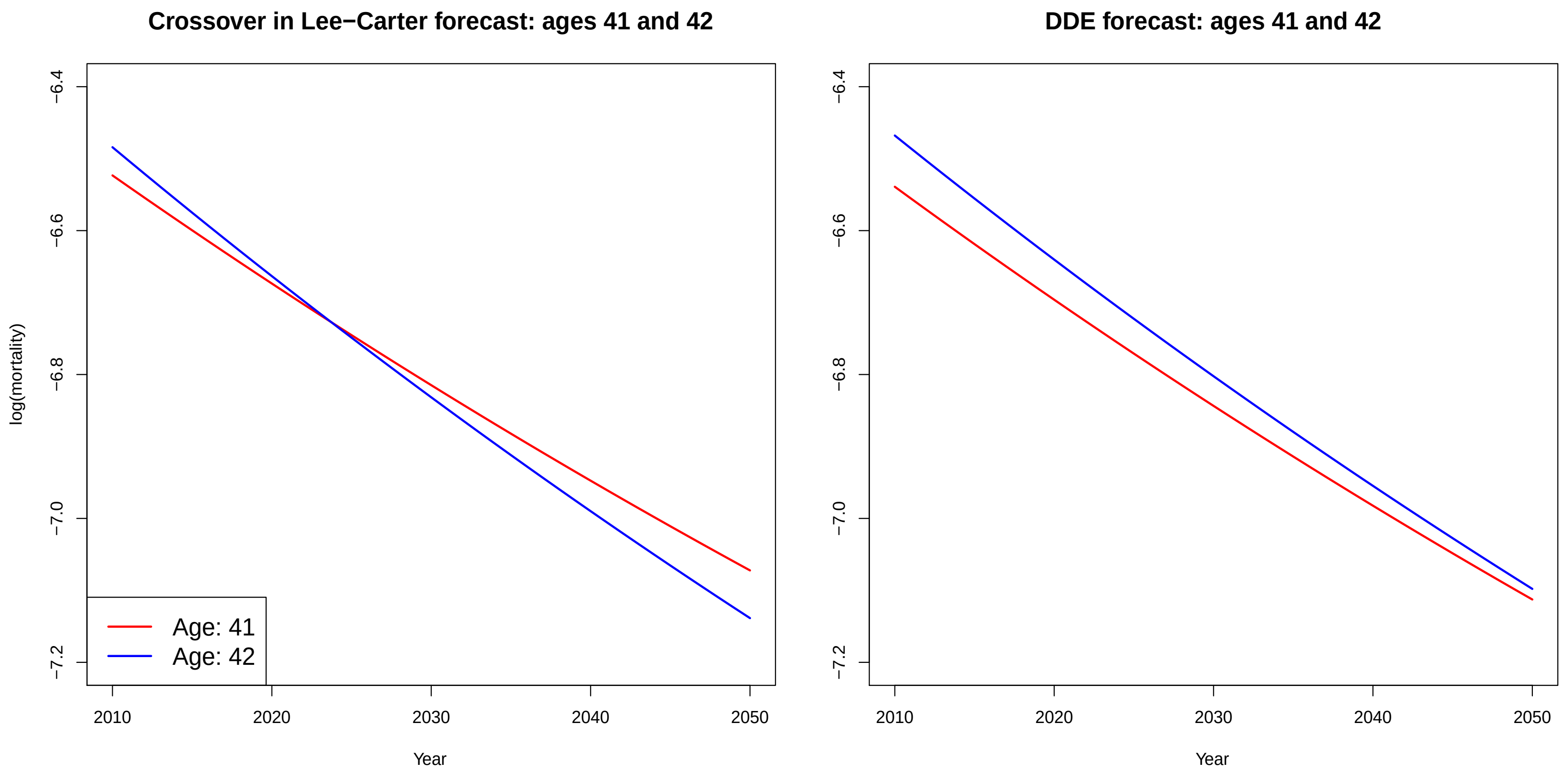
Their solution was

- Smooth the β_i with P-splines.

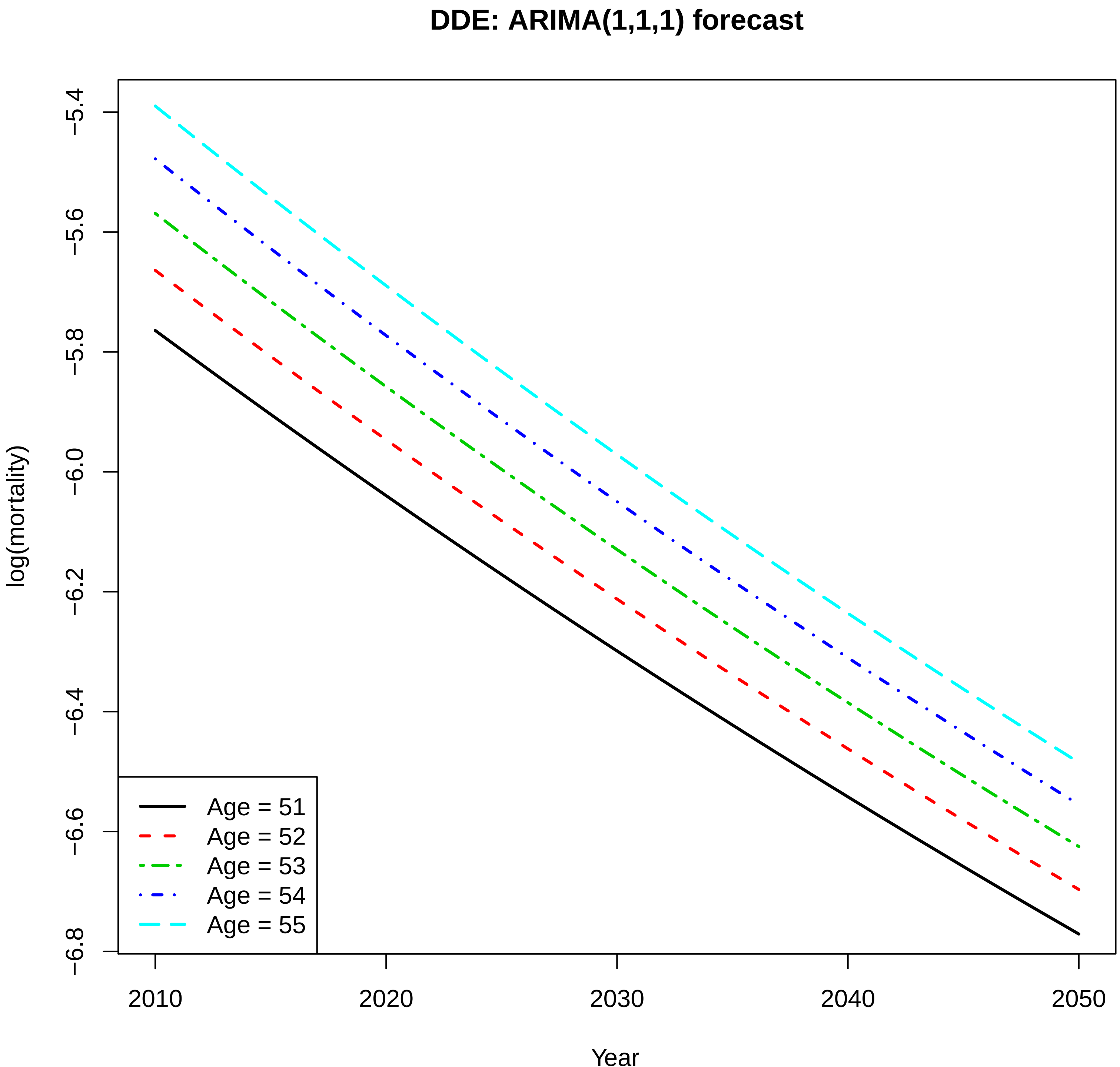
We denote their model by DDE.



LC
DDE
0.015
0.020
0.025
Beta
40
50
60
70
80
90
Age



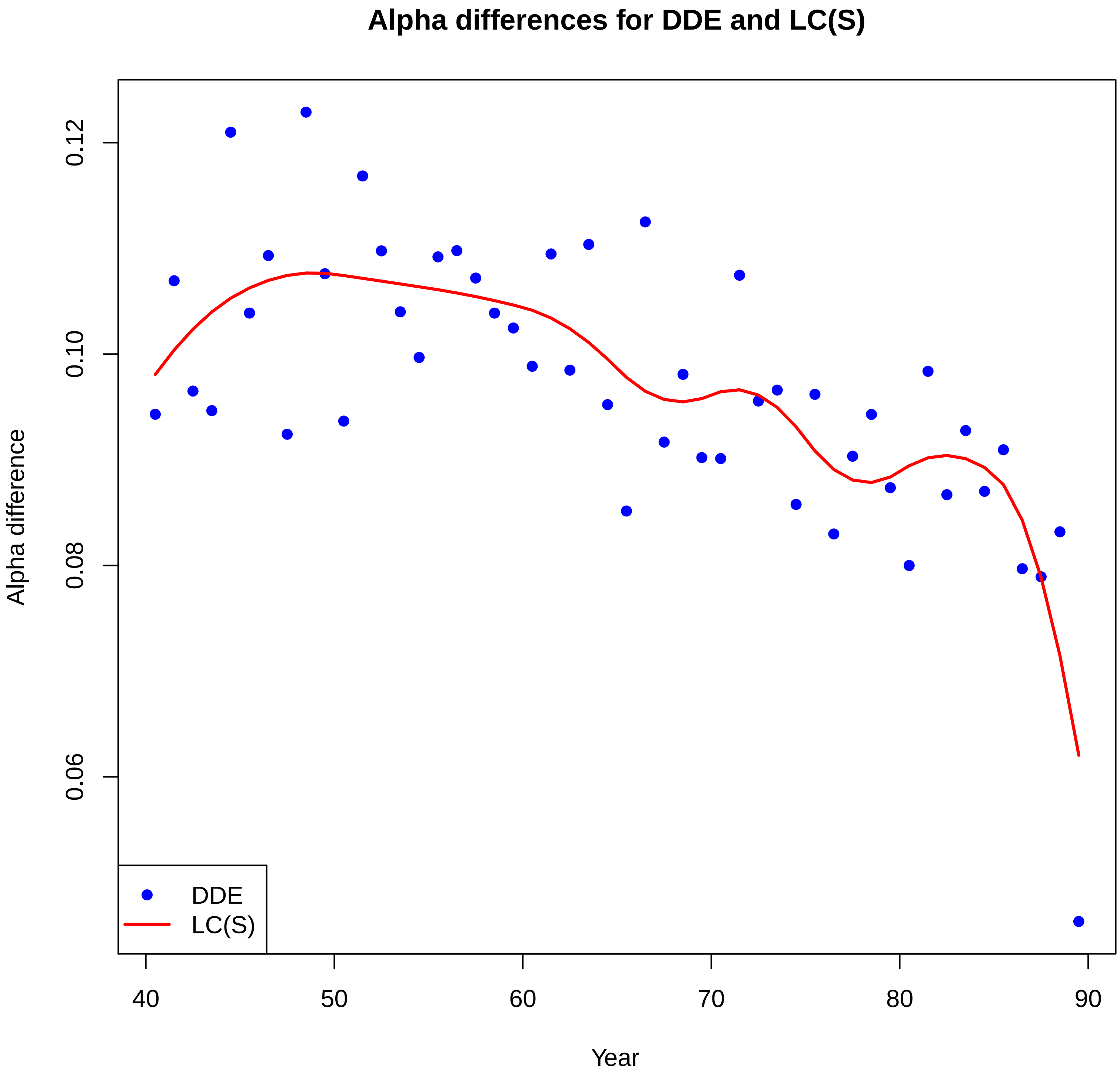
Conclusion: The DDE model does not suffer from crossovers to nearly the same extent as the Lee-Carter model. The next slide shows that the problem of irregularity with the Lee-Carter model is also much improved.


DDE: ARIMA(1,1,1) forecast
log(mortality)
Year
−5.4
−5.6
−5.8
−6.0
−6.2
−6.4
−6.6
−6.8
2010
2020
2030
2040
2050
Age = 51
Age = 52
Age = 53
Age = 54
Age = 55

Fine-tuning DDE

- Problem: Some irregularities in the forecasts remain.
- Solution: Smooth the α_i as well .

Refer to the LC model with smooth $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ as LC(S).



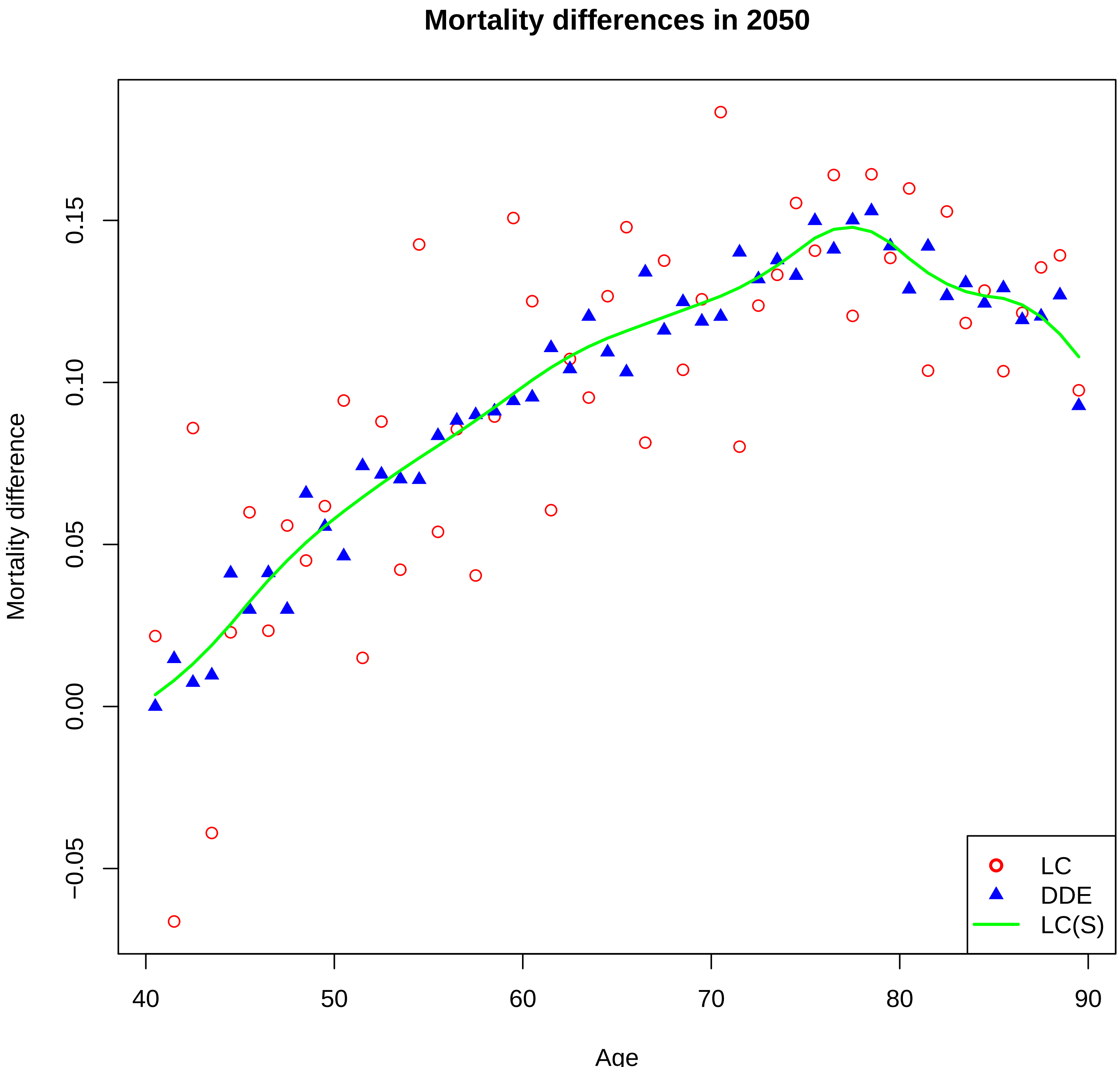
Alpha differences for DDE and LC(S)
Alpha difference
0.06
0.08
0.10
0.12
Year
40
50
60
70
80
90
DDE
LC(S)

Mortality Differences

The next slide shows mortality differences for adjacent ages in the final forecast year, ie,

$$\log(\hat{\mu}_{i+1,2050}) - \log(\hat{\mu}_{i,2050})$$

for ages $i = 40, \ldots, 89$. We want these differences to be smooth. The LC differences are quite irregular, the DDE differences are improved but only the LC(S) differences are fully smooth.


Mortality differences in 2050
Mortality difference
Age
0.15
0.10
0.05
0.00
−0.05
40
50
60
70
80
90
LC
DDE
LC(S)

Summary

- The Lee-Carter model is widely used, but suffers from drawbacks such as crossover and irregularity.
- The DDE model is a variant of LC which uses partial smoothing to reduce crossover and improve regularity.
- To fully minimize crossover and irregularity we use LC(S), a variant of LC which smooths both α and β.

References

- Lee & Carter (1992). J of American Statistical Association.
- Delwarde, Denuit & Eilers (2007). Statistical Modelling.
- The Lee-Carter family. Projections Toolkit Technical Guide, Chapter 9.